Example: The scale height of the Earth's atmosphere

We found that the equation for the scale height is

$$h_0 = \frac{RT}{m_m g}$$

Note that this assumes a constant temperature.

The Earth's atmosphere consists of approximately 80% N_2 and 20% O_2. (A most unusual and unstable combination!).

A mole consists of $N_\mathrm{A} = 6.0221 \times 10^{23}$ molecules. By definition, the weight of a mole in grams is equal to its atomic number:

- $m_m(\mathrm{H}) = 1.0\,\mathrm{g}$
- $m_m(\mathrm{C}) = 12.0\,\mathrm{g}$
- $m_m(\mathrm{O}) = 16.0\,\mathrm{g}$
- $m_m(\mathrm{O_2}) = 32.0\,\mathrm{g}$

The atomic number of 80% N_2 and 20% O_2 is $0.8 \times 28.0 + 0.2 \times 32.0 = 28.8$

The weight $m_m = 28.8 \times 10^{-3}\,\mathrm{kg}$

$R = 8.31\,\mathrm{J\,K^{-1}}$ is the universal gas constant.

$g = 9.8\,\mathrm{m\,s^{-2}}$

$T = 288\,\mathrm{K}$

Fill in, to find

$$h_0 = 8480\,\mathrm{m} = 8.5\,\mathrm{km}$$

In reality, the temperature drops a bit at higher altitude, to perhaps 250 K. This decreases the scale height to $\sim$ 7.5 km.

The atmosphere is extremely thin compared to the radius of the Earth.

How does the scale height compare to those of other planets?

- Venus: twin to Earth except for $T \sim 740\,\mathrm{K}$.
  $h_0 \approx 22\,\mathrm{km}$
- Mars: $g = 3.7\,\mathrm{m\,s^{-2}}$, $T \approx 220\,\mathrm{k}$
  $h_0 \approx 17\,\mathrm{km}$
- Pluto: $g = 0.66\,\mathrm{m\,s^{-2}}$, $T \approx 45\,\mathrm{k}$
  $h_0 \approx 20\,\mathrm{km}$
- Titan: $g = 1.35\,\mathrm{m\,s^{-2}}$, $T \approx 93\,\mathrm{k}$
  $h_0 \approx 20\,\mathrm{km}$

Note that this says nothing about the *density* of the atmospheres.